From Substitutes to Complements: Human Capital Allocation between Social and Technological Innovation

Felipe Chávez-Bustamante*

Cristián Troncoso-Valverde‡

Abstract

Firms face a critical trade-off when allocating scarce human capital between technological and social innovation. While the role of human capital in technological innovation is well established, its influence on social innovation and the strategic interplay between the two remain underexplored. We develop a two-stage game-theoretic model in which a firm first commits to social innovation, which differentiates the product and may generate cost-reducing spillovers to a rival, and then chooses how much human capital to invest in technological innovation that lowers its own marginal cost. The model delivers an endogenous switching mechanism whereby the strategic relationship between the two innovation types is not fixed but shifts from substitutability at low levels of commitment to complementarity once a critical threshold is surpassed. This shift is driven by the firm's ability to establish a sufficiently high degree of differentiation that softens competition, thereby amplifying the returns to cost reduction. Our findings are robust to N-firm Cournot and Bertrand price competition, thus providing a theoretical foundation for understanding complex innovation portfolios and offering a clear rationale for allocating human capital across innovation types.

Keywords: Social innovation, technological innovation, human capital, innovation strategy, strategic complementarities, resource allocation.

*Facultad de Economía, Negocios & Gobierno, Universidad San Sebastián. Email: felipe.chavez@uss.cl

†Facultad de Economía & Negocios, Universidad Andres Bello. Email: cristian.troncoso.v@unab.cl

‡Corresponding author.

1 Introduction

Sustaining a competitive advantage requires firms to make astute decisions about their innovation portfolios. This becomes particularly relevant when allocating scarce specialised human capital across two innovation paths: technological innovation, which drives private returns through productivity gains and cost reduction (Teece, 2007), and social innovation, which seeks to address societal challenges as a means to enhance brand perception and achieve product differentiation (Porter et al., 2006). The ability to strategically manage this allocation is therefore critical for balancing short-term efficiency with long-term market positioning and sustainable competitive advantage (Teece, 2007; O'Reilly III and Tushman, 2013).

While the link between human capital and technological innovation is well-documented, its role in driving social innovation —and the strategic trade-offs that arise when firms pursue both simultaneously— remains comparatively underexplored (Akgüç, 2020; Grimm et al., 2013; Unceta et al., 2016). As a result, the literature has yet to provide a formal model that formalises the economic dynamics that govern a firm's decision to allocate scarce talent between these competing paths. This paper addresses this theoretical gap by developing a game-theoretic model to answer three central questions: Under what conditions do investments in social and technological innovation act as strategic substitutes or complements? How does this strategic relationship depend on a firm's level of commitment to social innovation? How do market structure and the costs of specialized human capital moderate a firm's optimal innovation portfolio? Addressing these questions is fundamental to understanding the mechanism that can turn a costly innovation trade-off into a source of sustainable competitive advantage.

To answer these questions, we construct a two-stage game of strategic investment and market competition. In the first stage, a 'leader' firm commits to investing specialized human capital in social innovation, which enhances product differentiation but may generate cost-reducing spillovers for a rival. In the second stage, the firm chooses its investment in technological innovation to lower its own marginal cost.

The model's architecture, namely a pioneering firm investing in innovation with a fixed, sequential investment structure, is a deliberate choice designed to isolate the paper's core mechanism. The sequential timing reflects the economic reality that brand-building through social innovation is a long-term, strategic commitment (Oeij et al., 2019; Morsy et al., 2024), whereas technological improvements are more flexible, tactical decisions (March, 1991; Benner and Tushman, 2003). This leader-follower structure is therefore the necessary architecture to investigate how a firm's commitment to social innovation shapes its subsequent tactical choices on cost efficiency.

Our analysis reveals a novel endogenous switching mechanism whereby the strategic relationship between social and technological innovation is not fixed but shifts non-linearly with a firm's commitment to social innovation. Initially, the two investments act as strategic substitutes; however, once this commitment surpasses a critical threshold, they become powerful strategic complements. This shift is driven by the strategic logic of market competition. In the substitutes phase, the firm's differentiation is fragile, and knowledge spillovers that benefit a rival intensify competition, forcing a stark trade-off between cost efficiency and branding. In the complements phase, however, a deep investment in social innovation creates a "profit-margin multiplier." The resulting strong product differentiation insulates the firm from intense competition, making every unit of cost reduction from technological innovation more valuable and amplifying its impact on profit. We show that this fundamental mechanism is robust to variations in market structure (N-firm competition) and the mode of competition (Bertrand vs. Cournot), providing the first formal model to endogenise this complex dynamic.

This paper contributes to three distinct streams of literature. First and foremost, we contribute to the economic theory of innovation by providing a new application for the literature on strategic complementarities (Milgrom and Roberts, 1990; Cassiman and Veugelers, 2006). In the innovation literature, this framework has been used to analyse the synergy between different strategic choices, such as the well-documented complementarity between a firm's internal R&D efforts and its acquisition of external knowledge (Cassiman and Veugelers, 2006; Veugelers, 1997). Nonetheless, the focus of this literature has primarily been on static complementarities between established activities. Our paper contributes to this literature by formalizing a dynamic mechanism where the relationship between two innovation investments is not fixed but shifts endogenously from substitutability to complementarity. To our knowledge, this is the first formal model to endogenise such a shift, thereby providing a new theoretical micro-foundation for a complex, real-world strategic problem.

Our paper also contributes to the literature on human capital. It is well-established that human capital is a key driver of technological innovation, both at the national level for fostering economic growth and at the firm level for building absorptive capacity (Dakhli and De Clercq, 2004; Suseno et al., 2020; Lund Vinding, 2006; Sun et al., 2020). However, its role in driving social innovation and the strategic trade-offs that arise when firms pursue both simultaneously, remains comparatively underexplored (Akgüç, 2020; Grimm et al., 2013). We contribute by developing one of the first formal models to analyse the strategic allocation of scarce human capital between these two competing innovation paths, a question largely absent from the current literature.

Finally, our work connects to the growing literature on social innovation. This concept is distinct from corporate philanthropy, as it aims to create both societal and economic value by embedding solutions to social challenges within a firm's core strategy (Dionisio and de Vargas, 2020; Gasparin et al., 2021). From an economic perspective, social innovation can be viewed as a powerful mechanism for product differentiation and enhancing brand perception, thereby softening market competition (Porter et al., 2006; Dionisio and de Vargas, 2020). Although research in this area is expanding, much of the work remains qualitative or focused on identifying antecedents without modelling the competitive dynamics (Unceta et al., 2016; Van der Have and Rubalcaba, 2016). Our paper provides a formal, economic treatment of social innovation as a competitive strategy. Instead of treating it in isolation, we model its direct and indirect effects within an oligopolistic market, including its impact on a rival firm through knowledge spillovers. By doing so, we analyse its strategic interaction with traditional technological innovation, a dynamic that has been acknowledged but not previously formalized.

The remainder of the paper is organised as follows. Section 2 develops our baseline theoretical model and analyses the firm's strategic decisions in a duopoly. Section 3 extends the analysis to consider robustness to many competitors and to price competition. Section 4 discusses the broader implications of our findings, and Section 5 concludes.

2 The Baseline Model

This section develops our core theoretical framework. We begin by outlining the model's primitives (the production technology, consumer preferences, and innovation mechanisms), before proceeding to the equilibrium analysis.

2.1 Model Setup

We model a duopoly with two risk-neutral firms competing in a differentiated goods market. To isolate the strategic trade-offs central to our research questions, we model an innovating 'leader' firm (firm L) that strategically invests in innovation and a 'traditional' rival (firm E) that does not. This asymmetry between the firms allows us to analyse the emergence of social innovation as a competitive strategy, where a first-mover explores this new path while its competitor benefits only from potential knowledge spillovers. Both firms use a Cobb-Douglas production technology and serve a continuum of consumers with identical utility functions.

Firm L hires labour not only to produce its good but also to undertake two different types of innovation activities: technological and social. Technological innovations are aimed at enhancing firm L's own productivity through internal R&D, capabilities and resources. In contrast, social innovations seek to address broader societal problems, potentially creating new business opportunities by engaging a variety of actors (e.g., consumers, government agencies, competitors, non-profit organisations). This conceptualisation implies different consequences for the firm. Thus, while technological innovation efforts solely affect firm L's productivity, social innovation can generate positive productivity spillovers benefiting firm E and can also improve Firm L's brand perception among consumers, thereby increasing product differentiation.

The game unfolds in three stages. In the first stage, Firm L chooses its level of human capital investment in social innovation (h_s). In the second stage, it chooses its investment in technological innovation (h_p). The sequence is chosen to reflect the distinct economic and strategic nature of these investments: while investment in social innovation represents a long-term, hard-to-reverse strategic commitment to brand differentiation (Oeij et al., 2019; Morsy et al., 2024), investment in technological innovation is a more flexible, tactical decision on cost efficiency made within the context of an established corporate strategy (March, 1991; Benner and Tushman, 2003). All investment decisions become common knowledge before the third stage, in which firms compete in the product market. Following the investment stages, consumers observe prices and product characteristics and decide how much to consume of each product by maximising their utility subject to the usual budget constraints. The outcome of this maximisation process is a demand system relating quantities demanded to the vector of prices. Finally, given this demand structure, firms L and E engage in Cournot quantity competition in the product market.

To save on notation and whenever there is no risk of confusion, we use the notation $f'(x_0)$ and $f''(x_0)$ to denote the first and second derivative of f with respect to x evaluated at $x = x_0$.

2.1.1 Production, Innovation, and Consumer Preferences

Production. Each firm $j \in \{L, E\}$ uses a Cobb-Douglas technology,

$$q_j = A_j L_j^{\alpha} K_j^{1-\alpha},$$

where q_j is the quantity produced, A_j is total factor productivity, L_j and K_j are labour and capital inputs, and $\alpha \in (0, 1)$ is the output elasticity of labour. Firms are price takers in factor markets, and face prices $w > 0$ for labour and $r > 0$ for capital.

Technological Innovation. Firm L invests $h_p \in [0, \bar{h}]$ (where $\bar{h} > 0$ denotes the level at which A_L attains its maximum) units of specialised human capital to enhance its productivity $A_L(h_p)$. We normalise $A_L(0) = 1$ and assume that $A_L(h_p)$ is strictly increasing and strictly concave on $[0, \bar{h}]$.

Social Innovation. Firm L also invests $h_s \in [0, \bar{h}]$ units of specialised human capital in social innovation, which has two effects. First, it generates a negative externality for firm L in the form of knowledge spillovers that increase firm E's productivity, $A_E(h_s, \xi)$, where $\xi \in (0, 1)$ is the spillover intensity, and $A_E(0, \xi) = 1$.[1] We assume that the productivity functions $A_L(h_p)$ and $A_E(h_s, \xi)$ are strictly increasing and concave in h_p and h_s, respectively, and human capital is always more productive when applied internally to technological innovation than it is when its knowledge spills over to a rival.[2]

Second, investing human capital in social innovation creates a positive competitive effect for firm L by enhancing brand perception, which increases product differentiation. This is captured by the parameter $\sigma(h_s) \in [0, 1]$, which we assume is strictly decreasing in h_s, with $\sigma(0) = 1$ (homogeneous products) and $\sigma(\bar{h}) = 0$ (full differentiation) .

Consumers. Following Singh and Vives (1984), consumers are utility maximisers that choose quantities of firm L's good (q_L), firm E's good (q_E), and a numeraire good q_0, whose price is normalised to one. Consumers have the same quadratic utility function:

$$U(q_L, q_E, q_0) = V(q_L + q_E) - \frac{1}{2}(q_L^2 + q_E^2) - \sigma(h_s)q_L q_E + q_0$$

where $V > 0$ reflects market size and $\sigma(h_s)$ is the degree of product substitution, which decreases as differentiation increases.

Payoffs and Equilibrium. Firms' payoffs are determined through their strategic interaction in the product market, resulting from the cost and demand functions derived from their respective optimisation problems. In each case, the optimisation is carried out given the amounts of human capital (h_s, h_p) that firm L invests in social and technological innovation.

Let $p_j > 0$ denote the price of firm j's good. The consumers' optimisation problem yields the following linear demand structure (Amir et al., 2017; Nie and Yang, 2023; Singh and Vives, 1984):

$$p_j = V - q_j - \sigma(h_s)q_k, \quad j \neq k. \tag{1}$$

[1] For example, social innovation activities, which often involve external collaboration with a wide range of actors, can create unintended knowledge spillovers that benefit competitors, representing a key strategic risk for the innovating firm (Troncoso-Valverde and Chávez-Bustamante, 2024).

[2] Formally, this means that for all $(h_s, h_p) \in [0, \bar{h}]^2$ and $\xi \in (0, 1)$,

$$0 \le \frac{\partial \ln A_E(h_s, \xi)}{\partial h_s} \le \frac{\partial \ln A_L(h_p)}{\partial h_p}.$$

such that, per unit of specialised human capital, the percentage productivity gain that spills over to the rival does not exceed the percentage productivity gain from own technological investment.

where $\sigma(h_s) \in [0, 1]$. Likewise, firms' cost minimisation results in constant marginal costs of the form $c_j = \phi_j q_j$. Specifically, the marginal costs for firm L and firm E are:

$$\phi_L(h_p) = \frac{w^\alpha r^{1-\alpha}}{A_L(h_p)}\eta(\alpha) \quad \text{and} \quad \phi_E(h_s, \xi) = \frac{w^\alpha r^{1-\alpha}}{A_E(h_s, \xi)}\eta(\alpha)$$

where $\eta(\alpha)$ is the Cobb–Douglas constant arising from cost minimisation.[3]

Given the assumed increasing and concave properties of the productivity functions $A_L(\cdot)$ and $A_E(\cdot)$, the marginal cost $\phi_L(h_p)$ is strictly decreasing and convex in h_p, and $\phi_E(h_s, \xi)$ is strictly decreasing and convex in h_s.

Given these (inverse) demands and cost functions, firm L's and firm E's payoff functions are:

$$\pi_L(q_L, q_E) = [V - q_L - \sigma(h_s)q_E - \phi_L(h_p)]\, q_L - w_p h_p - w_s h_s$$

and,

$$\pi_E(q_L, q_E) = [V - q_E - \sigma(h_s)q_L - \phi_E(h_s, \xi)]\, q_E$$

where $w_s > 0$ and $w_p > 0$ are the unit costs of human capital for social and technological innovation, respectively. We assume V is sufficiently large to ensure positive equilibrium quantities. The solution concept is Subgame Perfect Nash Equilibrium.

2.2 Analysis

We analyse our model using backwards induction. Detailed mathematical derivations and formal proofs are provided in Appendix B.

2.2.1 Product Market Competition

The following result summarises competition in the product market given any prior investments in social and technological innovations. Mathematical derivations of equilibrium quantities and profits can be found in Appendix B.

Proposition 1. *For any given innovation investments (h_s, h_p), the product market subgame has a unique Nash equilibrium in which firms produce the following quantities:*

$$q_L(h_s, h_p) = \frac{2(V - \phi_L(h_p)) - \sigma(h_s)(V - \phi_E(h_s, \xi))}{4 - \sigma^2(h_s)} \tag{2}$$

and,

$$q_E(h_s, h_p) = \frac{2(V - \phi_E(h_s, \xi)) - \sigma(h_s)(V - \phi_L(h_p))}{4 - \sigma^2(h_s)} \tag{3}$$

[3]From cost minimisation under Cobb–Douglas technology, $\eta(\alpha) \equiv \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} + \left(\frac{1-\alpha}{\alpha}\right)^{\alpha}$.

and earn profits equal to:

$$\Pi_L(h_s, h_p) = \left[\frac{2(V - \phi_L(h_p)) - \sigma(h_s)(V - \phi_E(h_s, \xi))}{4 - \sigma^2(h_s)}\right]^2 - w_p h_p - w_s h_s$$

and,

$$\Pi_E(h_s, h_p) = \left[\frac{2(V - \phi_E(h_s, \xi)) - \sigma(h_s)(V - \phi_L(h_p))}{4 - \sigma^2(h_s)}\right]^2$$

Proposition 1 states the equilibrium quantities and profits for both firms given their strategic innovation investments. As expected (e.g. Vives, 1999), equilibrium quantities depend positively on market size (approximated by consumers' reservation utility V), and negatively on own costs. Note that our assumption on consumers' reservation utility, V, ensures strictly positive quantities in equilibrium for all $(h_s, h_p) \in [0, \bar{h}]^2$.

2.2.2 Differentiation and cost-reduction effects of social innovation

Investing in social innovation presents firm L with a strategic trade-off, which can be understood by examining the two channels through which h_s affects the firm's profit function, $\Pi_L(h_s, h_p)$. First, this investment enhances product differentiation, which can soften competition. Second, it creates a cost effect via knowledge spillovers that can intensify competition by reducing firm E's costs. The total effect is found by differentiating firm L's profit with respect to h_s, holding h_p fixed:

$$\frac{d\Pi_L(h_p, h_s)}{dh_s} =$$

$$\underbrace{\frac{\partial \Pi_L(h_p, h_s)}{\partial \phi_E}\frac{d\phi_E(h_s)}{dh_s}}_{\text{Cost effect}} + \underbrace{\frac{\partial \Pi_L(h_p, h_s)}{\partial \sigma}\frac{d\sigma(h_s)}{dh_s}}_{\text{Differentiation effect}} - w_s \tag{4}$$

where the terms labelled as *cost effect* and *differentiation effect* correspond to the previously described effects of social innovation on firm L's profits. Some algebra yields:

$$\frac{\partial \Pi_L(h_p, h_s)}{\partial \phi_E} = 2q_L(h_p, h_s)\left(\frac{\sigma(h_s)}{4 - \sigma^2(h_s)}\right) \geq 0$$

which, combined with $\phi_E(\cdot)$ being decreasing, confirms the intuition that social innovation negatively impacts firm L's profits because it reduces the costs for its competitors.

The analysis of the differentiation effect is more nuanced. As previously mentioned, a higher (or lower) degree of differentiation affects the nature of competition in the product market because it changes the degree of differentiation and, hence, consumers' willingness to pay for products from firm L's *and* firm E's products. As a result, the intensity with which firms compete in the product market changes, and so do firm L's profits. Differentiating firm L's profit function with respect to σ yields:

$$\frac{\partial \Pi_L(h_p, h_s)}{\partial \sigma} = \\ -2q_L(h_p, h_s)\left[\frac{(V-\phi_E(h_s))\sigma^2(h_s) - 4(V-\phi_L(h_p))\sigma(h_s) + 4(V-\phi_E(h_s))}{(4-\sigma^2(h_s))^2}\right]$$

Substituting the expressions for $q_L(h_s, h_p)$ and $q_E(h_s, h_p)$, this derivative can be simplified to:

$$\frac{\partial \Pi_L(h_p, h_s)}{\partial \sigma} = \frac{2q_L(h_s, h_p)}{4-\sigma^2(h_s)}[\sigma(h_s)q_L(h_s, h_p) - 2q_E(h_s, h_p)]$$

It is readily seen that for any given level of h_p, this partial derivative must be negative when evaluated at $\sigma = 0$. Because $\sigma = 0$ when $h_s = \bar{h}$, then this result suggests that for high amounts of human capital, the differentiation channel raises profits when h_s is large, which is unsurprising because $\sigma(\bar{h}) = 0$ is equivalent to fully differentiated products. In contrast, for low amounts of human capital invested in social innovation, $\sigma(\cdot)$ becomes closer to one, and therefore, the sign of this derivative depends on the magnitudes of $q_L(h_s, h_p)$ and $q_E(h_s, h_p)$. Some algebra yields:

$$q_L(0, h_p) - 2q_E(0, h_p) = \frac{\overbrace{4(V-\phi_L(h_p)) - 5(V-\phi_E(0,\xi))}^{\Psi(h_p)}}{3}$$

It is almost immediate that $\Psi(0) < 0$ due to the non-negativity conditions of Cournot quantities, and $\phi_L(0) = \phi_E(0, \xi)$. Moreover, because $\Psi(\cdot)$ is increasing due to $\phi_L(\cdot)$ being decreasing in h_p, we either have (i) $\Psi(h_p) < 0$ for all $h_p \in [0, \bar{h}]$, or (ii) $\Psi(h_p) < 0$ for $h_p \in [0, \hat{h}_p)$, and $\Psi(h_p) > 0$ for $h_p \in (\hat{h}_p, \bar{h}]$.

From expression (4) and the fact that $\sigma'(h_s) < 0$, the differentiation effect encourages the investment of human capital in socially oriented innovations if, and only if, $\frac{\partial \Pi_L}{\partial \sigma} < 0$. This requires $\Psi(h_p) < 0$, which is more likely to hold when the level of human capital in technological innovation is relatively low. This suggests that the incentives to invest in social innovation are tied to the *level* of investment in technological innovation and consequently, that the firm's allocation of human capital to socially oriented innovations depends not only on market responses but also on the firm's internal decisions related to R&D and innovation investments. This insight is crucial, as it establishes that the firm's optimal social innovation strategy cannot be determined in isolation; it is inextricably linked to its technological innovation strategy.

2.2.3 Social innovation as a long-term commitment

As mentioned earlier, social innovation often requires years to manifest because it aims to address complex societal challenges that require shifts in behaviours, norms, and social structures. This longer timeline for social innovation stands in contrast to technological innovations, which generally have a more immediate focus and operate on shorter timelines with quicker market applications (Dahlin and Behrens, 2005; Jacobsson and Bergek, 2004; Kelly et al., 2021).

Given a first-stage investment in social innovation $h_s \in [0, \bar{h}]$, firm L chooses its second-stage investment in technological innovation, h_p, to solve the following maximization problem:

$$\max_{h_p \in [0,\bar{h}]} \Pi_L(h_s, h_p) \tag{5}$$

As the profit function is continuous and the choice set is compact, a solution to this problem exists. We denote the set of solutions by $\mathcal{H}_p(h_s)$, with typical element:

$$h_p^*(h_s) = \underset{h_p \in [0,\bar{h}]}{\text{argmax}}\ \Pi_L(h_s, h_p)$$

The following lemma provides sufficient conditions involving the consumers' reservation utility V, and the minimal return to the initial investment in technological innovations such that $\mathcal{H}_p(h_s)$ is a singleton with $h_p^*(\cdot) > 0$.

Lemma 1. *Let $\mathcal{H}_p(h_s)$ denote the set of solutions to firm L's problem (5). If V satisfies:*

$$V > 3\phi_L(0) \tag{6}$$

then $\mathcal{H}_p(h_s)$ is a singleton, $h_s \in [0, \bar{h}]$. Moreover, if in addition to condition (6), the following condition also holds:

$$A'_L(0) > \frac{4w_p}{\phi_L(0)[V - 2\phi_L(0) + \phi_E(\bar{h}, \xi)]} \tag{7}$$

then $h_p^*(\cdot) > 0$, for all levels of $h_s \in [0, \bar{h}]$.

Lemma 1 provides conditions under which firm L actively invests in cost reduction after committing to any level of human capital allocated to social innovation activities. The critical question, however, is how this optimal investment, h_p^*, varies with the first-stage commitment to social innovation, h_s. If, for instance, these investments are strategic complements, then achieving greater differentiation enhances the rewards from cost efficiency, potentially justifying a strategy that pursues investments in both areas simultaneously. Conversely, if these investments are strategic substitutes, then firm L faces a choice between cost efficiency and developing a long-term relationship with customers that leads to higher brand differentiation, as attempting both may be less effective than focusing resources on just one. Thus, the relationship between h_p and h_s alters the marginal incentives for different types of innovation, forcing the firm to consider not only the direct benefits from investing in each type of innovation but also their crucial strategic interaction when making its initial choice of h_s.

The following Proposition identifies a critical strategic threshold: human capital investments in social and technological innovation initially act as substitutes, with increased differentiation through social innovation reducing marginal returns from cost reductions. However, once differentiation surpasses a critical threshold, these investments become complementary, mutually enhancing the firm's competitive advantage. The proof of this proposition is in Appendix B.

Proposition 2. *Let conditions in Lemma 1 hold. There exist some threshold levels of human capital, h_l and h_u, with $0 < h_l \leq h_u < \bar{h}$, such that, when h_p is set to its optimal value $h_p^*(h_s)$, human capital invested in technological and social innovation behaves as substitutes for $h_s \in [0, h_l)$, and as complements for $h_s \in (h_u, \bar{h}]$.*

This shift from substitution to complementarity captures a fundamental dynamic in strategic innovation investment, providing a clear answer to our central research questions. Initially, at low levels of commitment to social innovation, the two investment paths act as strategic substitutes, driven by both resource contention (as a limited pool of specialised human capital means investment in technology comes at the direct opportunity cost of social initiatives) and the vulnerability of early-stage differentiation, where the negative effect of knowledge spillovers to a more cost-efficient rival can diminish the firm's nascent branding advantage. The firm thus faces a stark choice between two competing paths: a marginal gain in internal efficiency or a marginal gain in external differentiation.

However, as the commitment to social innovation surpasses a critical threshold, the strategic logic inverts, and the two investment types become strategic complements. A high level of h_s is not just a marginal improvement; it represents a strategic transformation of the firm's market position. The firm has now established a strong, differentiated brand identity, effectively insulating it from intense price competition. This differentiation acts as a profit-margin multiplier; that is, every unit of cost reduction from technological innovation becomes more valuable as it is applied to a product that commands a higher price premium. The strong brand identity thus acts as a shield, protecting the returns from technological investment while amplifying their impact on the bottom line. This synergy, where strong differentiation makes cost efficiency more valuable, is the key to unlocking superior, long-term competitive advantage.

This transition from a trade-off to a synergy provides a clear theoretical explanation for how strategic commitment can unlock complex, non-linear dynamics in a firm's innovation portfolio.

We now turn to the firm's first-stage problem: choosing the optimal commitment to social innovation, h_s, anticipating its effect on the second-stage choice, $h_p^*(h_s)$. Firm L's problem is to solve:

$$\max_{h_s \in [0,\bar{h}]} \Pi_L(h_s, h_p^*(h_s)) \tag{8}$$

An interior solution, h_s^*, must balance the marginal benefit of investment against its marginal cost, w_s. Differentiating the profit function with respect to h_s gives the marginal benefit, $MB_s(h_s)$:

$$MB_s(h_s) = \frac{2q_L(h_s)}{4-\sigma^2(h_s)}\left[\sigma(h_s)\phi_E'(h_s) + [\sigma(h_s)q_L(h_s) - 2q_E(h_s)]\sigma'(h_s)\right]$$

where, for simplicity, we write $q_L(h_s) \equiv q_L(h_s, h_p^*(h_s))$ and similarly for q_E.

The sign of the marginal benefit, $MB_s(h_s)$, is ambiguous because it captures two opposing effects: the positive impact of increased differentiation (since $\sigma' < 0$) and the negative impact of cost-reducing spillovers to the rival (since $\phi_E' < 0$). The overall sign depends on the term $[\sigma(h_s)q_L(h_s) - 2q_E(h_s)]$. At the boundaries, the sign of this term is determinate: it is negative at full differentiation ($h_s = \bar{h}$) and positive at zero differentiation ($h_s = 0$), provided that $V < 5\phi_E(0) - 4\phi_L(h_p^*(0))$.

Lemma 2. *Define $\mathcal{H}_s = \{h_s \in (0, \bar{h}] : MB_s(h_s) = w_s\} \cup \{\bar{h}\}$. If $V < 5\phi_E(0) - 4\phi_L(h_p^*(0))$ and $w_s > \bar{w}_s$, where*

$$\bar{w}_s = \sup_{h_s \in H_s} \left\{ \frac{q_L^2(h_s) - q_L^2(0) - w_p(h_p^*(h_s) - h_p^*(0))}{h_s} \right\}$$

then the unique solution to firm L's problem (8) is $h_s^* = 0$.

Intuitively, the conditions in Lemma 2 are likely to hold in markets characterised by demands with small intercept V or with a shortage of human capital (which reflects in high salaries). In such markets, resource-constrained firms face a trade-off with respect to the allocation of human capital: they prioritise technological innovation to maximise profits because a reduction in the investments of human capital allocated to social innovation activities not only increase benefits by itself but also frees up resources for technological oriented innovations, which leverage firm L's competitive advantage through a lower production cost. In this context, setting $h_s = 0$ reflects a strategic choice driven by the substitute nature of social and technological innovations, as well as market constraints.

The case of markets characterised by demands with sufficiently large intercepts still requires a minimal degree of differentiation to induce firm L to invest human capital in social innovation that justifies the cost of allocating human capital to this type of activity.

Lemma 3. *Suppose that (i) $V > 5\phi_E(0) - 4\phi_L(h_p^*(0))$, (ii) $w_s < \bar{w}_s$ and,*

$$\text{(iii) } |\sigma'(0)| > \frac{9w_s + 6q_L(0)|\phi'(0)|}{2q_L(0)(V - 5\phi_E(0) + 4\phi_L(h_p^*(0)))}$$

hold. Then, any solution to firm L's problem (8) must involve a strictly positive amount of human capital invested in social innovation.

Lemma 3 describes the conditions under which social innovation becomes a viable strategy In large markets, with not too high wage ($w_s < \bar{w}_s$), and a high enough initial differentiation rate ensure that the strategic advantages of increased differentiation is sufficient to outweigh not only the direct cost associated to this investment but also the negative impact of cost-reducing spillovers to firm E. The optimal amount of human capital invested in socially oriented innovations must balance this trade-off, which occurs at the point where the marginal gains from extra differentiation equal the marginal losses from a more efficient competitor.

3 Extensions and Robustness

3.1 Competition with many firms

To understand how market structure moderates our central findings, we now extend the baseline model to analyse competition dynamics in markets where firm L faces $N - 1$ identical competitors ($N \geq 2$). We maintain the core structure, namely, firm L is the only firm investing human capital (sequentially) in social and technological innovation, and all competitors produce identical substitutes for firm E's (with similar cost function as that of firm E's). To ensure tractability, we assume all rival firms are perfect substitutes for one another, yielding a demand function for firm L of $p_L = V - q_L - \sigma(h_s)\sum_{j\neq L} q_j$. Firms compete a la Cournot in the product market.

Similar to the baseline model, competition in the product market yields firm L profits equal to:

$$\Pi_L(h_s, h_p, N) = [q_L(h_s, h_p, N)]^2 - w_p h_p - w_s h_s \tag{9}$$

where,

$$q_L(h_s, h_p, N) =$$

$$\frac{(2 + 2\sigma(h_s)(N-2))(V - \phi_L(h_p)) - \sigma(h_s)(N-1)(V - \phi_E(h_s))}{4 + 4\sigma(h_s)(N-2) - \sigma^2(h_s)(N-1)} \tag{10}$$

Simple inspection reveals that the behaviour of firm L's profit regarding the number of competitors in the market (given h_s and h_p) depends on how $q_L(\cdot)$ changes as N does so. The following lemma formalises this analysis.

Lemma 4. *For any fixed* $(h_s, h_p) \in [0, \bar{h}]^2$*, firm* L*'s profit,* $\Pi_L(h_s, h_p, N)$*, is non-increasing in the number of firms in the market* N*. Furthermore,* $\Pi_L(h_s, h_p, N)$ *is strictly decreasing in* N *if* $h_s \in (0, \bar{h})$*.*

In the second stage, firm L chooses h_p to maximise its profit. For an interior solution to exist, the marginal benefit of the first unit of investment must exceed its cost, w_p, which requires:

$$A_L'(0) > \frac{w_p}{2\phi_L(0) q_L(h_s, 0, N) \left| \frac{\partial q_L(0, h_s, N)}{\partial \phi_L} \right|} \tag{11}$$

The right-hand side of this expression depends on N through the product between $q_L(0, h_s, N)$ and $\left| \frac{\partial q_L(0, h_s, N)}{\partial \phi_L} \right|$. This product decreases with N because firm L is not initially more efficient than the rest of the competitors (which holds at $h_p = 0$ as $\phi_L(0) \geq \phi(h_s, \xi)$; see Lemma 9 in Appendix A). Consequently, the right-hand side of (11) increases with N. Thus, as competition intensifies, condition (11) becomes harder to satisfy, making positive investment in h_p less likely and potentially driving h_p^* towards zero in highly competitive markets.

Now suppose that condition (11) holds. We can examine the *direct* effect (that is, keeping h_s constant) of an increase in N on the incentives to invest in technological innovations by means of the first-order condition (FOC) associated with the optimal choice of h_p. Rearranging terms in this first-order condition yields,

$$\left| \phi_L'(h_p^*) \right| = \frac{w_p}{2 z_s(h_p^*, N)} \tag{12}$$

where $z_s(h_p, N)$ denotes the product between $q_L(h_s, h_p, N)$ and (the absolute value of) $\frac{\partial q_L(h_s, h_p, N)}{\partial \phi_L}$ (which, in the case of (12) is evaluated at the optimal h_p^*). The effect of an increase in N on the optimal h_p^* then depends on how $z_s(h_p^*, N)$ changes with N.

Lemma 5. *Consider any history in which firm L invests an amount $h_s \in [0, \bar{h}]$ of human capital in socially oriented innovations. An increase in the number of competitors in the market, N,*

(i) reduces firm L's incentives to invest human capital in technological innovations if this firm is already the most inefficient (or equally efficient) firm in the market;

(ii) induces an ambiguous effect on the optimal investment h_p in response to changes in N if firm L is not the most inefficient firm in the market.

Lemma 5 reveals that increased competition (a higher N) affects firm L's investment in technological innovation differently, based on its relative efficiency. Thus, if firm L is the most inefficient firm in the market, then a higher N decreases its incentive to invest in h_p^* because its shrinking market share (q_L^*) against more efficient rivals diminishes the returns from investing in this type of innovation. Conversely, if firm L is initially the most efficient firm (which is plausible if spillovers from h_s are not too strong), then the impact of a higher N on h_p^* is ambiguous: while fiercer competition might eventually erode the incentive for a higher investment, it still is possible that firm L can leverage cost reductions for greater market share due to a higher responsiveness of quantity to cost changes in more crowded markets.

We now examine how the strategic relationship between h_s and h_p changes with market structure. Under some technical conditions (see Lemma 10 in Appendix A), the following lemma shows that the complementarity result from our baseline model is robust: for any number of competitors, the two innovation paths become strategic complements when the commitment to social innovation is sufficiently high.

Lemma 6. *For any $N \geq 2$, there exists a threshold value $\hat{h}_u < \bar{h}$ such that $\frac{\partial^2 \Pi_L(h_p, h_s, N)}{\partial h_p \partial h_s} > 0$ for all $h_s \in [\hat{h}_u, \bar{h}]$. Thus, for all h_s sufficiently high, investments of human capital in technological and socially oriented innovations behave as strategic complements.*

The analysis of the strategic interaction near $h_s = 0$ is more complex because a marginal increase in h_s from zero triggers two opposing effects: the initial differentiation softens competition, potentially raising the marginal value of cost reduction through h_p, whereas the reduction in the rivals' costs intensifies competition, potentially lowering the marginal profitability of reducing firm L's cost. Consequently, the behaviour of h_s and h_p near $h_s = 0$ depends on the net outcome of these two opposing forces. The next lemma provides sufficient conditions under which investments of human capital in social and technological innovation act as strategic substitutes for any finite N near $h_s = 0$.

Lemma 7. *Let $\mathcal{H}_p(0,N)$ be the set of all solutions to firm L's problem when $h_s = 0$:*

$$\mathcal{H}_p(0,N) = \left\{ h_p^* \in [0,\bar{h}] : h_p^* \in \underset{h_p \in [0,\bar{h}]}{\operatorname{argmax}} \ \Pi_L(h_p, 0, N) \right\}$$

If there exists some $h_p^* \in \mathcal{H}_p(0,N)$ such that $h_p^* > 0$ and

$$\frac{V - \phi_L(h_p^*)}{V - \phi_E(0,\xi)} > \frac{N+3}{2N} \tag{13}$$

holds, then there exists some $\hat{h}_l > 0$ such that $\frac{\partial^2 \Pi_L(h_p,h_s,N)}{\partial h_p \partial h_s} < 0$ for all $h_s \in [0,\hat{h}_l]$. Thus, under the previous two conditions, investments of human capital in technological and socially oriented innovations behave as strategic substitutes for h_s close to zero.

Observe that condition (13) trivially holds for all $N \geq 4$ because $\phi_L(h_p) \leq \phi_E(0,\xi)$ implies that the left-hand side of this condition is weakly greater than one, whereas the right-hand side becomes strictly less than one when $N \geq 4$.

Intuition suggests that firm L's incentives to differentiate diminish as N increases because fiercer competition pushes this firm's market share down. Thus, despite the potentially positive gains from cost efficiency (see Lemma 5), the reduced attractiveness of higher differentiation should drive h_p and h_s to behave as strategic substitutes when h_s is small. This contrasts with scenarios involving high levels of commitment in socially oriented innovations (see Lemma 6), where firm L can achieve a significant market power by increasing investments in both h_s and h_p irrespective of the number of competitors in the market.

Lemma 8. *There exists some N^{**} such that for all $N > N^{**}$ there is some $\hat{h}_l > 0$ such that $\frac{\partial^2 \Pi_L(h_s,h_p,N)}{\partial h_p \partial h_s} < 0$ for all $h_s \in [0,\hat{h}_l]$.*

Lemma 8 suggests that cost-based competition predominates over differentiation-based strategies in highly competitive markets, at least when firm L's commitment to social innovation is low, even though differentiation is the primary aim of engaging in socially oriented activities.

In the first stage, firm L chooses the optimal investment of human capital in socially oriented innovations by solving:

$$\max_{h_s \in [0,\bar{h}]} \Pi_L(h_s, h_p^*(h_s,N), N) \tag{14}$$

where $h_p^*(h_s,N)$ is its optimal second-stage choice. While closed-form solutions for the solution to this problem are not readily available, positive solutions ($h_s^* > 0$) require the marginal benefit of investing h_s when h_s is arbitrarily close to zero, to exceed its marginal cost:

$$\frac{\partial q_L^*(h_p^*(0),0,N)}{\partial h_s} > \frac{w_s}{2q_L^*(h_p^*(0),0,N)} \tag{15}$$

where $\frac{\partial q_L(0,h_p^*(0),N)}{\partial h_s}$ condenses the marginal effects of h_s on differentiation (via $\sigma'(0)$) and on the competitors' costs (via $A_E'(0)$). Interestingly, if condition (15) holds then,

$$\frac{V-\phi_L(h_p^*)}{V-\phi(0,\xi)} < \frac{N+3}{2N}$$

which is the reverse of condition (13) in Lemma 7, where we established substitutability between h_p and h_s near $h_s = 0$. Therefore, $h_s^* > 0$ arises precisely when the strategic relationship near $h_s = 0$ is ambiguous, which allows for potential complementarity between h_s and h_p. Intuitively, positive investments in h_s^* occur if the initial benefit accruing from differentiation outweighs the negative effect of more efficient competitors. Such a scenario, potentially involving initial complementarity, is plausible if firm L lacks a strong efficiency advantage, making even slight competitive softening from h_s significantly profitable.

The previous results suggest that while our core substitute-to-complement dynamic persists, intense market competition makes the initial 'substitution trap' more difficult to escape, requiring an even stronger strategic commitment to social innovation to achieve synergy.

To analyse the firm's incentives to commit to social innovation in a highly competitive market, we examine the limiting case as the number of firms goes to infinity. Let $h_s^*(N)$ be an optimal solution to (14), and define firm L's limiting profit function as $N \to \infty$ by $\Pi_L^*(h_s)$. Standard conditions allow the interchange of limit and maximisation operations, yielding:[4]

$$\begin{aligned}\Pi_L^*(h_s) &= \lim_{N\to\infty}\left\{\max_{h_p\in[0,\bar{h}]}\left[(q_L(h_s,h_p,N))^2 - w_p h_p\right]\right\} \\ &= \max_{h_p\in[0,\bar{h}]}\left\{\lim_{N\to\infty}\left[(q_L(h_s,h_p,N))^2 - w_p h_p\right]\right\}\end{aligned} \tag{16}$$

Proposition 3. *Let $\Pi_L^*(h_s)$ be given by (16). There exists a threshold wage, $w_s^* > 0$, defined by,*

$$w_s^* = \frac{\Pi_L^*(\bar{h}) - \Pi_L^*(0)}{\bar{h}}$$

such that for all $w_s < w_s^*$ any sequence of optimal choices $h_s^*(N)$ converges to $\bar{h}$ as $N \to \infty$,

$$\lim_{N\to\infty} h_s^*(N) = \bar{h}$$

[4] Because h_p belongs to the compact set $[0,\bar{h}]$, the objective function is continuous in h_p for any given N, $\lim_{N\to\infty} q_L(h_s,h_p,N)$ exists for all $h_s \in [0,\bar{h}]$ and $q_L(h_s,h_p,N)$ converges to this limit as N grows large, the limit of the maximum value must be equal to the maximum value of the limit function,

$$\lim_{N\to\infty}\left(\max_{h_p\in[0,\bar{h}]}\Pi_L(h_s,h_p,N)\right) = \max_{h_p\in[0,\bar{h}]}\left(\lim_{N\to\infty}\Pi_L(h_s,h_p,N)\right)$$

whereas for all $w_s \geq w_s^*$ *any sequence of optimal choices* $h_s^*(N)$ *converges to* 0 *as* $N \to \infty$*,*

$$\lim_{N\to\infty} h_s^*(N) = 0$$

Proposition 3 reveals potential specialisation as N grows large, as partial differentiation yields profits from intermediate levels of investment not higher than those at $h_s = 0$ under intense competition. Thus, firm L is forced to choose between achieving complete differentiation (i.e., choosing $h_s = \bar{h}$) or completely abandoning social innovation ($h_s = 0$), a decision that depends on the cost w_s.

This relationship with w_s also determines the strategic interplay between h_s and h_p in the limit. If a high value of w_s forces investment in social innovation to tend to zero, then h_s and h_p will behave as strategic substitutes near this equilibrium. Conversely, if a low value of w_s leads this investment to $\bar{h}$, then h_s and h_p will behave as strategic complements (Lemma 6), supporting potentially positive investments in technological innovation (i.e., $h_p^* > 0$). This suggests that the cost of hiring specialised human capital, w_s, is a critical determinant of both the social innovation level and its strategic interaction with technological innovation in highly competitive markets.

3.2 Price competition

To ensure our central finding, i.e., the shift from substitutes to complements, is not an artifact of a specific competitive mode, we now test the model's robustness by assuming firms compete on price (Bertrand) rather than quantity. The sequential investment structure remains unchanged.[5]

Firms $j = L, E$ choose prices p_j to maximise profits given by $\pi_j = (p_j - \phi_j)q_j(p_L, p_E)$, where quantities q_j are derived from inverting the demand system (1). A key distinction from Cournot's model of competition is the discontinuity of these profits at $h_s = 0$, where products become perfect substitutes. Indeed, standard Bertrand competition with identical costs (which occurs when $h_s = h_p = 0$) yields $p_L^* = p_E^* = \phi_L(0)$ and zero profits. Alternatively, if $h_p > 0$ (so that $\phi_L(h_p) < \phi_E(0, \xi)$), then firm L's cost advantage allows it to capture the entire market by setting $p_L^*(h_p, 0) = \phi_E(0, \xi)$, and earning profits equal to:

$$\Pi_L(h_p, 0) = [\phi_E(0, \xi) - \phi_L(h_p)](V - \phi_E(0, \xi)) - w_p h_p - w_s h_s$$

whereas firm E earns zero profits.[6]

[5] Our Cournot setup can be viewed as a reduced-form for capacity competition, followed by price setting when capacity is costly. See Chapter 5 in Tirole (1988) for a discussion about reduced-form profits like the ones we employ, resulting from short-run price competition with given capacities.

[6] We adopt the standard tie–breaking rule that assigns all demand to the lowest–price (lowest–cost) firm when products are undifferentiated; see Blume (2003).

Finally, after histories in which products are differentiated (i.e., if $h_s > 0$), Bertrand competition yields equilibrium prices:

$$p_L^B(h_p, h_s) = \frac{V(1-\sigma(h_s))(2+\sigma(h_s)) + 2\phi_L(h_p) + \sigma(h_s)\phi_E(h_s,\xi)}{4-\sigma^2(h_s)} \tag{17}$$

and the corresponding symmetric expression for p_E^B. Firm L's profits are:

$$\Pi_L^B(h_p, h_s) = \frac{(p_L^B - \phi_L(h_p))^2}{1-\sigma^2(h_s)} - w_p h_p - w_s h_s$$

which can be conveniently rewritten as follows:

$$\Pi_L^B(h_p, h_s) = [q_L^B(h_p, h_s)]^2(1-\sigma^2(h_s)) - w_p h_p - w_s h_s$$

where $q_L^B(h_p, h_s)$ is the equilibrium quantity that results from firms L and E setting prices equal to p_L^B and p_E^B, respectively.

Momentarily consider price competition in a setup where investments in social innovation are strictly positive. Differentiating firm L's relevant profit function yields:

$$\frac{\partial \Pi_L^B(h_p, h_s)}{\partial h_p} = \frac{2\phi_L(h_p)A_L'(h_p)}{(4-\sigma^2(h_s))A_L(h_p)}\left[q_L^B(h_p, h_s)(2-\sigma^2(h_s))\right] - w_p \tag{18}$$

Alternatively, the FOC from Cournot's competition is:

$$\frac{\partial \Pi_L^C(h_p, h_s)}{\partial h_p} = \frac{2\phi_L(h_p)A_L'(h_p)}{(4-\sigma^2(h_s))A_L(h_p)}\left[2q_L^C(h_p, h_s)\right] - w_p \tag{19}$$

where we have used the superscript C to highlight the fact that this derivative corresponds to a market in which firms compete a la Cournot. Note that the difference between these two FOCs lies in the bracketed terms. Let $\zeta(h_p) = (2-\sigma^2(h_s))q_L^B - 2q_L^C$. Some algebra yields,

$$\zeta(h_p) = \frac{\sigma(h_s)^3[\phi_E(h_s) - \phi_L(h_p)\sigma(h_s) - V(1-\sigma(h_s))]}{(1-\sigma(h_s)^2)(4-\sigma(h_s)^2)}$$

and therefore, the sign of $\zeta(h_p)$ (and hence, the incentives to invest in technological innovations) depends on whether $\sigma(h_s)$ is greater or lower than some threshold $\hat{\sigma}(h_p)$,

$$\hat{\sigma}(h_p) = \frac{V - \phi_E(h_s,\xi)}{V - \phi_L(h_p)}$$

This comparison extends to the conditions required to ensure a positive investment in h_p. Let A_L^B and A_L^C denote the threshold levels under Bertrand and Cournot competition, respectively, required to induce a positive level of investments in h_p,

$$A_L^{Bd} = \frac{w_p(4-\sigma^2(h_s))}{2(2-\sigma^2(h_s))q_L^B(0,h_s)\phi_L(0)}; \qquad A_L^C = \frac{w_p(4-\sigma^2(h_s))}{4q_L^C(0,h_s)\phi_L(0)}$$

When $h_s > 0$ (i.e., when products are differentiated) and $h_p = 0$, firm L has no initial cost advantage and hence, $\phi_L(0) \geq \phi_E(h_s, \xi))$, from where it follows $\hat{\sigma}(0) > 1$. Thus, $\sigma(h_s) < \hat{\sigma}(0)$, leading to a higher threshold ($A_L^B > A_L^C$) for initial investments in h_p under Bertrand competition compared to Cournot competition. This suggests that the less intense nature of quantity competition provides a stronger baseline incentive for cost-reducing investment when products are already differentiated.

The analysis of the case of homogeneous products (i.e., under $h_s = 0$) is similar. Differentiating the relevant profit function and evaluating this derivative at $h_p = 0$ yields the following condition:

$$A'_L(0) > \underbrace{\frac{w_p}{\phi_L(0)(V-\phi_E(0,\xi))}}_{A_L^{Bh}}$$

It is immediate that Bertrand's competition makes it easier to initiate technological innovations when products are homogeneous.

Proposition 4. *Given $h_s > 0$, let $h_p^B(h_s)$ and $h_p^C(h_s)$ be interior solutions to firm L's problem at stage two under Bertrand and Cournot competition, respectively. If $\sigma(h_s) \leq \hat{\sigma}(h_p^B)$, then $h_p^B(\cdot) \leq h_p^C(\cdot)$, i.e., Bertrand competition induces weakly lower investment in h_p. If $\sigma(h_s) > \hat{\sigma}(h_p^C)$, then $h_p^B(\cdot) > h_p^C(\cdot)$, i.e., Bertrand competition induces higher investment in h_p. Given $h_s = 0$, Bertrand competition induces higher investment in h_p.*

Proposition 4 summarizes these findings, showing that the intensity of price competition has a non-monotonic effect on the incentive to invest in technological innovation. Bertrand competition dampens investment when products are differentiated but amplifies it when products are homogeneous.

The following proposition confirms that our central finding is robust to the mode of competition in the product market. The non-linear relationship between social and technological innovation persists under price competition, shifting from strategic substitutes to strategic complements as the commitment to social innovation deepens.

Proposition 5. *Assume that $A'_L(0) > A_L^B$ and condition (20) in Lemma 11 holds. Then, under Bertrand competition there exist thresholds h_l and h_u, $0 < h_l \leq h_u < \bar{h}$, such that $h_p^B(h_s)$ and h_s are strategic substitutes for $h_s \in (0, h_l)$ and strategic complements for $h_s \in [h_u, \bar{h}]$.*

We now analyse firm L's optimal first-stage commitment to social innovation under price competition. This choice has important consequences for the firm's innovation path and the resulting market structure. The simplest scenario is one where firm L forgoes innovation entirely (i.e., $h_s^* = h_p^* = 0$). This occurs when two conditions are met: (i) the initial return to technological innovation is too low to be profitable, and (ii) the cost of specialized human capital, w_s, is prohibitively high,

$$w > \bar{w}_s = \sup_{h_s \in (0,\bar{h}]} \left\{ \frac{\bar{\Pi}_L^B(h_s, h_p^*(h_s)) - \bar{\Pi}_L^B(0, h_p^*(0))}{h_s} \right\}$$

where $\bar{\Pi}_L^B(h_s, h_p^*(h_s))$ is firm L's profit function (before subtracting $w_s h_s$) after histories in which $h_s > 0$, and $\bar{\Pi}_L^B(0, h_p^*(0))$ this firm's profits after histories in which $h_s = 0$.[7]

The previous configuration is, perhaps, more likely to arise in technologically mature markets where consumers are not particularly sensitive to social or communal problems, or where the costs associated with implementing any type of innovation are exceptionally high relative to the potential competitive gains that they offer. In contrast, markets where consumers place a significant value on ethical production, sustainability, and other social values, and where brand reputation is a key source of differentiation, are more likely to provide the correct incentives for investing in socially oriented activities, provided that specialised human capital is affordable.

Proposition 6. *If $w_s < \bar{w}_s$, then committing to some (positive) degree of social innovation is optimal.*

The crucial element behind Proposition 6 is the idea that it is the affordability of specialised human capital that enables social innovation as a viable, profit-enhancing strategy for the firm. The threshold $\bar{w}_s$ captures the maximum additional value (per unit of h_s) that social innovation can bring compared to forgoing it entirely, even after considering any associated technological innovations. Thus, according to this proposition, if the cost of investing in human capital w_s is below this potential incremental gain, investing in this type of activity becomes profitable. Incidentally, note that common social initiatives (such as circular economy models or waste reduction programs) can also lead to cost savings, thereby increasing the threshold value $\bar{w}_s$ and making the condition $w_s < \bar{w}_s$ more likely to hold. Similarly, the availability of subsidies/incentives for socially responsible practices can effectively lower the net cost of socially oriented initiatives, which would be captured by $\bar{w}_s$. This analysis ultimately confirms that the fundamental shift from strategic substitutes to complements is a robust phenomenon, holding under both price and quantity competition, although the

[7]Because $\phi_L(0) = \phi_E(0, \xi)$ the threshold wage $\underline{w}_s$ is finite because $\frac{\Pi_L^B(h_s, h_p^*(0))}{h_s} = O(1)$ as $h_s \to 0^+$.

specific incentives for technological investment are moderated by the type of product market competition involved.

4 Discussion

Our findings offer important implications for theory, policy, and practice. The model's primary theoretical contribution is to the literature on strategic complementarities and innovation strategy, as well as on innovation ambidexterity. While previous work has established how clusters of reinforcing activities can create value, it has primarily focused on static synergies. Our model extends this by providing a formal mechanism that explains the non-linear dynamics of innovation that managers encounter when balancing potentially exploratory social innovations with the often more exploitative drives for technological efficiency.

The assumptions concerning sequential investment timing and the asymmetric firm structure play a central role in the analysis. Modelling social innovation as a long-term commitment that precedes the more flexible decision of technological investment allows us to formally analyse how a firm's foundational strategic shapes its subsequent tactical investments. Thus, at low levels of commitment the logic of resource contention and spillovers dominates, making the investments strategic substitutes, whereas at high levels, the logic of market power and strong differentiation from social innovation dominates, creating sufficient product differentiation that insulates the firm from competition and transforming the relationship into one of strategic complements. This finding goes beyond simply identifying a new context for complementarities; it formalizes the process by which they can emerge from an initial state of substitutability. Likewise, the asymmetric firm structure, with one innovating leader and one traditional rival, is a theoretical choice that allows us to cleanly analyse the strategic impact of knowledge spillovers without the confounding effects of bidirectional investments that would arise in a symmetric game. This allows our model to provide a clear micro-foundation for how social innovation can emerge as a competitive strategy in an otherwise conventional market.

Policy implications. These theoretical insights also offer policy and practical guidance. For policymakers, our analysis suggests that firms, especially those with constrained R&D budgets, may underinvest from a societal perspective in socially oriented activities, even when possessing the requisite human capital. Our model suggests that firms often do not internalise the positive externalities or navigate the typically longer, more uncertain pathways associated with social innovations that aim to modify established behaviours. Consequently, policies aimed at bridging the gap by lowering the firm's perceived risk and net cost can incentivise the use of human capital for broader societal benefit. Furthermore, our model's identification of a 'complementarity threshold' offers a guide for policy design, suggesting that broad, one-off subsidies for social innovation may be insufficient and that initial grants or innovation vouchers could be more helpful in fostering social innovations.

Strategic implications. For managers, our findings provide insights for those seeking to generate social value alongside private profit through the pursuit of innovation strategies that incorporate social innovation activities. Our theoretical analysis reveals a non-linear strategic interplay between social and technological innovation investments. This relationship is not static: while initial or low-level social commitments might act as substitutes for technological efforts, achieving a certain threshold of social innovation can transform this dynamic into one of complementarity, where both types of innovation reinforce each other. Understanding this non-linear relationship between substitutability and complementarity is crucial because achieving synergy may require a substantial initial commitment to social initiatives. Strategic resource allocation must also carefully consider the firm's external competitive environment and its internal cost structure to effectively balance these innovation paths.

Limitations and future research. We acknowledge that our model relies on simplifying assumptions to isolate the core mechanisms of interest. These assumptions define the boundaries of our contribution and simultaneously open several exciting avenues for future research.

First, our model assumes an asymmetric game (with one leader who is the only firm investing in innovation activities) with a fixed, sequential investment structure. This was a deliberate choice to analyse the emergence of social innovation as a pioneering strategy and to capture its nature as a long-term commitment. However, future theoretical work could relax these assumptions. For instance, one could model a symmetric game where the choice to pursue social innovation is endogenous for all firms, or explore simultaneous investment decisions to understand contexts where innovation paths are chosen concurrently.

Second, our model generates sharp, testable predictions that invite future empirical work. Our core finding, namely the shift from substitution to complementarity, could be tested by examining whether the interaction term between social and technological innovation investments becomes more positive for firms with stronger brand reputations or higher market differentiation. Furthermore, our extensions suggest that in highly competitive markets, firms are more likely to specialize in either a fully differentiated or a low-cost strategy, a prediction that could be tested using market structure data.

Finally, our model highlights the importance of institutional context. Future work could explore how factors like government subsidies, intellectual property regimes, or consumer sensitivity to social issues affect the "complementarity threshold" we identify. Such extensions could build upon our foundational findings to create a rich research program at the intersection of innovation strategy, public policy, and corporate social responsibility.

5 Conclusions

This paper develops a theoretical framework to analyse the strategic allocation of specialized human capital between social and technological innovation. Our model reveals that the strategic relationship between these two investment types is not fixed; they initially act as strategic substitutes but can transition to become strategic complements once investment in social initiatives surpasses a critical threshold. This core finding is robust to different forms of market competition, highlighting the stability of the underlying mechanism. Ultimately, by formalizing this dynamic, our paper establishes a foundation for understanding how firms can manage their innovation portfolios. It highlights that the key strategic question is not whether to pursue social or technological goals, but how to manage the commitment needed to shift their relationship from a costly trade-off into a source of sustainable competitive advantage.

Use of generative AI tools. During manuscript preparation we used Gemini 2.5 (Google) exclusively for language editing (grammar/clarity) on text written by the authors. The tool was not used to generate research content, derivations, proofs, or analysis. We reviewed and edited all suggestions and take full responsibility for the content of this manuscript.

References

Akgüç, M. (2020). Understanding the determinants of social innovation in europe: an econometric approach. *European Planning Studies*, 28(5):941–954.

Amir, R., Erickson, P., and Jin, J. (2017). On the microeconomic foundations of linear demand for differentiated products. *Journal of Economic Theory*, 169:641–665.

Benner, M. J. and Tushman, M. L. (2003). Exploitation, exploration, and process management: The productivity dilemma revisited. *Academy of management review*, 28(2):238–256.

Blume, A. (2003). Bertrand without fudge. *Economics Letters*, 78(2):167–168.

Cassiman, B. and Veugelers, R. (2006). In search of complementarity in innovation strategy: Internal r&d and external knowledge acquisition. *Management science*, 52(1):68–82.

Dahlin, K. B. and Behrens, D. M. (2005). When is an invention really radical?: Defining and measuring technological radicalness. *Research policy*, 34(5):717–737.

Dakhli, M. and De Clercq, D. (2004). Human capital, social capital, and innovation: a multi-country study. *Entrepreneurship & regional development*, 16(2):107–128.

Dionisio, M. and de Vargas, E. R. (2020). Corporate social innovation: A systematic literature review. *International business review*, 29(2):101641.

Gasparin, M., Green, W., Lilley, S., Quinn, M., Saren, M., and Schinckus, C. (2021). Business as unusual: A business model for social innovation. *Journal of Business Research*, 125:698–709.

Grimm, R., Fox, C., Baines, S., and Albertson, K. (2013). Social innovation, an answer to contemporary societal challenges? locating the concept in theory and practice. *Innovation: The European Journal of Social Science Research*, 26(4):436–455.

Jacobsson, S. and Bergek, A. (2004). Transforming the energy sector: the evolution of technological systems in renewable energy technology. *Industrial and corporate change*, 13(5):815–849.

Kelly, B., Papanikolaou, D., Seru, A., and Taddy, M. (2021). Measuring technological innovation over the long run. *American Economic Review: Insights*, 3(3):303–320.

Lund Vinding, A. (2006). Absorptive capacity and innovative performance: A human capital approach. *Economics of innovation and New Technology*, 15(4-5):507–517.

March, J. G. (1991). Exploration and exploitation in organizational learning. *Organization science*, 2(1):71–87.

Milgrom, P. and Roberts, J. (1990). The economics of modern manufacturing: Technology, strategy, and organization. *The American Economic Review*, pages 511–528.

Morsy, M., Trott, P., and Cho, D. S. (2024). Corporate social innovation: a systematic literature review and research agenda. *Society and Business Review*, 19(3):523–550.

Nie, P. and Yang, Y. (2023). Innovation and competition with human capital input. *Managerial and Decision Economics*, 44(3):1779–1785.

Oeij, P. R., Van Der Torre, W., Vaas, F., and Dhondt, S. (2019). Understanding social innovation as an innovation process: Applying the innovation journey model. *Journal of Business Research*, 101:243–254.

O'Reilly III, C. A. and Tushman, M. L. (2013). Organizational ambidexterity: Past, present, and future. *Academy of management Perspectives*, 27(4):324–338.

Porter, M. E., Kramer, M. R., et al. (2006). The link between competitive advantage and corporate social responsibility. *Harvard business review*, 84(12):78–92.

Singh, N. and Vives, X. (1984). Price and quantity competition in a differentiated duopoly. *The Rand journal of economics*, pages 546–554.

Sun, X., Li, H., and Ghosal, V. (2020). Firm-level human capital and innovation: Evidence from china. *China Economic Review*, 59:101388.

Suseno, Y., Standing, C., Kiani-Mavi, R., and Jackson, P. (2020). National innovation performance: the role of human capital and social capital. *Innovation: The European Journal of Social Science Research*, 33(3):296–310.

Teece, D. J. (2007). Explicating dynamic capabilities: the nature and microfoundations of (sustainable) enterprise performance. *Strategic management journal*, 28(13):1319–1350.

Tirole, J. (1988). *The theory of industrial organization*. MIT press.

Troncoso-Valverde, C. and Chávez-Bustamante, F. (2024). Do you want to know a secret? strategic alliances and competition in product markets. *European Journal of Operational Research*, 313(3):1180–1190.

Unceta, A., Castro-Spila, J., and García Fronti, J. (2016). Social innovation indicators. *Innovation: The European Journal of Social Science Research*, 29(2):192–204.

Van der Have, R. P. and Rubalcaba, L. (2016). Social innovation research: An emerging area of innovation studies? *Research Policy*, 45(9):1923–1935.

Veugelers, R. (1997). Internal r & d expenditures and external technology sourcing. *Research policy*, 26(3):303–315.

Vives, X. (1999). *Oligopoly pricing: old ideas and new tools*. MIT press.

Appendix A

This appendix contains some results that are not included in the main text.

Lemma 9. *Let* $z_p(N) = q_L(N)\left|\frac{\partial q_L(N)}{\partial \phi_L}\right|$*, where* $q_L(N)$ *is the equilibrium quantity in* (10)*. For any given* $h_s \in [0, \bar{h}]$ *and any* $h_p \in [0, \bar{h}]$ *such that* $\phi_L(h_p) \geq \phi_E(h_s, \xi)$*, the function* $z_p(N)$ *is non-increasing in* N *for* $N \geq 2$.

Proof of lemma 9. To simplify notation, let $\Delta z_p(N) = z_p(N+1) - z_p(N)$, and $a(N) = \left|\frac{\partial q_L(N)}{\partial \phi_L}\right|$. We want to show that $\Delta z_p(N) \leq 0$ under the condition $\phi_L(h_p) \geq \phi_E(h_s, \xi)$.

Note that we can equivalently write $\Delta z_p(N)$ as follows:

$$\Delta z_p(N) = q_L(N+1)\Delta a(N) + a(N)\Delta q_L(N)$$

Moreover, from the proof of lemma 4, $\Delta q_L(N) \leq 0$ and hence,

$$\begin{aligned}\Delta z_p(N) &\leq q_L(N)\Delta a(N) + a(N)\Delta q_L(N) \\ &= \frac{2\sigma(h_s)(1-\sigma(h_s))}{D(N+1)D(N)}\left\{\sigma(h_s)q_L(N) + a(N)[\sigma(V - \phi_L(h_p)) - 2(V - \phi(h_s,\xi))]\right\}\end{aligned}$$

where $D(k)$ is the denominator of $q_L(k)$ (Cournot's equilibrium quantity), $k \geq 2$.

Define

$$S(x) = xq_L(N) + a(N)[x(V - \phi_L(h_p)) - 2(V - \phi(h_s,\xi))]$$

It is immediate that $S(0) \leq 0$ (because $a(N) > 0$, $N \geq 2$, and $V > \phi_E(\cdot)$), and $S'(x) > 0$. Moreover,

$$\begin{aligned}S(1) &= q_L(N) + a(N)[(V - \phi_L(h_p)) - 2(V - \phi_E(h_s,\xi))] \\ &= \frac{[4 + 4\sigma(h_s)(N-2)](V - \phi_L) - (5\sigma N - 7\sigma + 4)(V - \phi_E)}{D(N)}\end{aligned}$$

Because the denominator of this expression is (strictly) positive, the sign of $S(1)$ depends on the sign of its numerator. Let $\phi_L(h_p) \geq \phi_E(h_s, \xi)$. Then $V - \phi_L(h_p) \leq V - \phi_E(h_s, \xi)$ and because $[4 + 4\sigma(h_s)(N-2)] > 0$ for all $N \geq 2$, it must be true that

$$\begin{aligned}S(1) &\leq [4 + 4\sigma(h_s)(N-2) - 5\sigma N + 7\sigma - 4](V - \phi_E(h_s,\xi)) \\ &= -\sigma(h_s)(N+1)(V - \phi_E(h_s,\xi)) \\ &\leq 0\end{aligned}$$

and thus, $S(x) \leq 0$ for all $x \in [0,1]$, and strictly so for $x \in (0,1)$. Because $q_L(N) > 0$ and $a(N) > 0$ for all $N \geq 2$, if $\phi_L(h_p) \geq \phi_E(h_s, \xi)$ then,

$$\Delta z_p(N) \leq \frac{2\sigma(h_s)(1-\sigma(h_s))}{D(N+1)D(N)}S(\sigma) \leq 0$$

with the last inequality strict if $\sigma \in (0,1)$, which is the desired result. □

Lemma 10. *Let*

$$\bar{V} = \frac{3\phi_L(h_p^*)[2+\sigma(N-2)] - 2\sigma(N-1)\phi_E(h_s,\xi)}{2(2-\sigma(h_s)}$$

For any given $h_s \in [0,\bar{h}]$, if $V > \bar{V}$ then $\Pi_L(h_s,h_p,N)$ is strictly concave with respect to $h_p \in [0,\bar{h}]$.

Proof of lemma 10. Differentiation of $\Pi_L(h_s,h_p,N)$ yields,

$$\frac{\partial^2 \Pi_L}{\partial h_p^2} = 2\left|\frac{\partial q_L}{\partial \phi_L}\right| e\left\{A_L'(h_p)g'(h_p) + A_L''(h_p)g(h_p)\right\}$$

where,

$$\left|\frac{\partial q_L}{\partial \phi_L}\right| = \frac{2+\sigma(N-2)}{4+2\sigma(N-2)-\sigma^2(N-1)}, \quad e = w^\alpha r^{1-\alpha}\eta(\alpha),$$

and, $g(h_p) = \frac{q_L(h_s,h_p,N)}{A_L^2(h_p)}$. Clearly, $g(h_p) > 0$, $A_L' > 0$ and $A_L'' < 0$ and hence, $\frac{\partial^2\Pi_L}{\partial h_p^2} < 0$ if $g'(h_p) \leq 0$. Some algebra yields,

$$g'(h_p) = \frac{A_L'(h_p)}{A_L^3(h_p)}\left[\left|\frac{\partial q_L}{\partial \phi_L}\right|\phi_L(h_p) - 2q_L(h_s,h_p,N)\right]$$

The term within square brackets is strictly negative provided that $V > \bar{V}$, which completes the proof. □

Lemma 11. *Define $\bar{V}$ as follows:*

$$\bar{V} = \frac{3(2-\sigma^2(h_s))\phi_L(0) - 2\sigma(h_s)\phi_E(h_s,\xi)}{2(1-\sigma(h_s))(2+\sigma(h_s))} \tag{20}$$

If $V > \bar{V}$ then $\Pi_L^B(\cdot,h_s)$ is a strictly concave function of h_p, for any $h_s \in (0,\bar{h}]$.

Proof of lemma 11. The proof is verbatim to the proof of Lemma 10 and hence omitted. □

Appendix B

This appendix contains the formal derivations and proofs of the results summarised in the main text.

Proof of proposition 1. Given Firm L's previous choices of human capital (h_s, h_p), firms L and E simultaneously choose quantities q_L and q_E to maximise their respective profits:

$$\pi_L(q_L, q_E) = [V - q_L - \sigma(h_s)q_E - \phi_L(h_p)]\, q_L - w_p h_p - w_s h_s$$

and,

$$\pi_E(q_L, q_E) = [V - q_E - \sigma(h_s)q_L - \phi_E(h_s, \xi)]\, q_E$$

taking their competitor's quantity as given. The best response functions are:

$$q_L(q_E) = \frac{V - \phi_L - \sigma q_E}{2} \quad \text{and} \quad q_E(q_L) = \frac{V - \phi_E - \sigma q_L}{2}$$

It is easily seen that, under our assumptions, these best response functions are downward-sloping, thereby yielding a unique pair of Nash equilibrium quantities for this continuation game. Solving the resulting system of equations yields the quantities in the proposition. Substituting these quantities in firms' profits in the text delivers $\Pi_L(h_s, h_p)$ and $\Pi_E(h_s, h_p)$ in the proposition. □

Proof of lemma 1. Differentiating $\Pi_L(h_s, h_p)$ with respect to h_p yields:

$$\frac{\partial \Pi_L(h_p, h_s)}{\partial h_p} = \left(\frac{4w^{\alpha} r^{1-\alpha}\eta(\alpha)}{4 - \sigma^2(h_s)}\right)\left(\frac{q_L(h_p, h_s)}{A_L^2(h_p)}\right)\frac{\partial A_L(h_p)}{\partial h_p} - w_p \tag{21}$$

Denote $g(h_p) \equiv \left(\frac{q_L(h_p,h_s)}{A_L^2(h_p)}\right)$ such that:

$$\frac{\partial^2 \Pi_L(h_p, h_s)}{\partial h_p^2} = \frac{4w^{\alpha} r^{1-\alpha}\eta(\alpha)}{4 - \sigma^2(h_s)}\left[\frac{\partial g(h_p)}{\partial h_p}\frac{\partial A_L(h_p)}{\partial h_p} + g(h_p)\frac{\partial^2 A_L(h_p)}{\partial h_p^2}\right] \tag{22}$$

Because $V > 3\phi_L(0) > 2\phi_E(h_s, \xi) - \phi_L(\bar{h})$ (which follows from non-negativity of Cournot quantities) then $q_L(\cdot, h_s) > 0$. Likewise, assumptions in the model guarantee that A_L, A_L', and g are strictly positive for all $h_p \in [0, \bar{h}]$, and that $A_L'' < 0$. Hence, the second term within square brackets must be non-positive. Furthermore,

$$\frac{\partial g(h_p)}{\partial h_p} = \frac{2}{A_L^2(h_p)}\left[\frac{\phi_L(h_p)}{4 - \sigma^2(h_s)} - q_L(h_p, h_s)\right]\frac{\partial A_L(h_p)}{\partial h_p}$$

where $A_L(h_p)\phi_L(h_p) = w^{\alpha} r^{1-\alpha}\eta(\alpha)$, and,

$$\begin{aligned}\frac{\phi_L(h_p)}{4-\sigma^2(h_s)} - q_L(h_p,h_s) &= \frac{3\phi_L(h_p) - 2V + \sigma(h_s)V - \sigma(h_s)\phi_E(h_s,\xi)}{4-\sigma^2(h_s)} \\ &\leq \frac{3\phi_L(0) - V}{4-\sigma^2(h_s)} \\ &< 0\end{aligned}$$

because $\phi_L(\cdot) \leq \phi_L(0)$, and $\sigma(\cdot) \in [0,1]$. Therefore, $\frac{\partial g(\cdot)}{\partial h_p}$ is strictly negative, which implies that $\Pi(\cdot, h_s)$ must be strictly concave provided that $V > 3\phi_L(0)$ and hence, the solution to firm L's problem must be unique.

Second, by contradiction, suppose that the conditions in the lemma hold and that there exists some $h_s \in [0, \bar{h}]$ such that $h_p^*(h_s) \leq 0$. Because condition (6) holds, then $h_p^*(h_s) = 0$ must be the unique solution to firm L's problem. However,

$$\begin{aligned}\left.\frac{\partial \Pi_L(h_p,h_s)}{\partial h_p}\right|_{h_p=0} &= \left(\frac{4\phi_L(0)}{4-\sigma^2(h_s)}\right) q_L^*(h_s,0)A_L'(0) - w_p \\ &> \frac{4q_L(h_s,0)}{4-\sigma^2(h_s)} q_L(h_s,0)\left(\frac{4w_p}{\phi_L(0)[V-2\phi_L(0)+\phi_E(\bar{h},\xi)]}\right) - w_p \\ &\geq \left(\frac{2(V-\phi_L(0)) - \sigma(h_s)(V-\phi_E(h_s,\xi))}{V-2\phi_L(0)+\phi_E(\bar{h},\xi)}\right) w_p - w_p \\ &\geq 0\end{aligned}$$

where the first inequality follows from replacing condition (7) into the above expression, the second from $4 - \sigma^2(h_s) \leq 4$, and the last one from $\phi_E(h_s,\xi) \geq \phi_E(\bar{h},\xi)$, and $V > \phi_E(\cdot,\xi)$. This contradicts $h_p^*(h_s) = 0$ being a solution to firm L's problem, and hence, $h_p^*(\cdot) > 0$ for any $h_s \in [0, \bar{h}]$. □

Proof of proposition 2. Suppose that both conditions in Lemma 1 hold such that firm L's profit function is strictly concave in h_p. By the implicit function theorem, the derivative of h_p^* with respect to h_s can be computed as follows:

$$\frac{\partial h_p^*(h_s)}{\partial h_s} = -\frac{\frac{\partial^2 \Pi(h_s, h_p^*(h_s))}{\partial h_s \partial h_p}}{\frac{\partial^2 \Pi(h_s, h_p^*(h_s))}{\partial h_p^2}} \tag{23}$$

Given the strict concavity of Π_L, the sign of (23) depends on the sign of the cross-partial derivative:

$$\frac{\partial^2 \Pi_L(h_s,h_p)}{\partial h_s \partial h_p} =$$

$$\frac{4\,|\phi_L'(h_p)|}{(4-\sigma^2(h_s))^2}\left[\sigma(h_s)\phi_E'(h_s)+\sigma'(h_s)[\sigma(h_s)q_L(h_p,h_s)(2\sigma^2(h_s)+1)-2q_E(h_p,h_s)]\right]$$

where the expressions for $q_L(\cdot)$ and $q_E(\cdot)$ are given by (2) and (3) respectively.

Note that the sign of this cross-partial derivative depends on whether the expression within square brackets is positive or negative. Evaluating this cross-partial derivative at $h_s = \bar{h}$ yields,

$$\left.\frac{\partial^2\Pi_L(h_s,h_p)}{\partial h_s\partial h_p}\right|_{h_s=\bar{h}} = \frac{|\phi_L'(h_p)|}{2}\left[-\sigma'(\bar{h})q_E(h_s,h_p)\right] > 0$$

where the inequality follows from $\sigma' < 0$ for all $h_s \in [0,\bar{h}]$. Likewise, evaluating the cross-partial derivative at $h_s = 0$ yields:

$$\left.\frac{\partial^2\Pi_L(h_s,h_p)}{\partial h_s\partial h_p}\right|_{h_s=0} = \frac{4\,|\phi_L'(h_p)|}{9}\left[\phi_E'(0)+\sigma'(0)[3q_L(0,h_p)-2q_E(0,h_p)]\right] < 0$$

because $3q_L(0,h_p) > 2q_E(0,h_p)$. By continuity of this cross-partial derivative and the Intermediate Value Theorem, there must exist some $h_s^* \in (0,\bar{h})$ such that $\frac{\partial^2\Pi_L(h_s^*,h_p^*(h_s^*))}{\partial h_s\partial h_p} = 0$. Let $h_l = \inf\{h_s \in (0,\bar{h}) : \frac{\partial^2\Pi_L}{\partial h_s\partial h_p} = 0\}$ and $h_u = \sup\{h_s \in (0,\bar{h}) : \frac{\partial^2\Pi_L}{\partial h_s\partial h_p} = 0\}$. It is immediate that $0 < h_l \le h_u < \bar{h}$. Moreover, $\frac{\partial^2\Pi_L(h_s,h_p^*(h_s))}{\partial h_s\partial h_p} < 0$ for $h_s \in [0,h_l)$, and $\frac{\partial^2\Pi_L(h_s,h_p^*(h_s))}{\partial h_s\partial h_p} > 0$ for $h_s \in (h_u,\bar{h}]$. Hence, h_s and h_p are substitutes for $h_s \in [0,h_l)$, and as complements for $h_s \in (h_u,\bar{h}]$. □

Proof of lemma 2. From the text,

$$MB_s(0) = \frac{2q_L(0)}{3}\left[\phi_E'(0)+[q_L(0)-2q_E(0)]\sigma'(0)\right]$$

Because $V < 5\phi_E(0) - 4\phi_L(h_p^*(0))$ then,

$$\sigma(0)q_L(0) - 2q_E(0) = \frac{5\phi_E(0)-4\phi_L(h_p^*(0))-V}{3} > 0$$

and $MB_s(0) - w_s < 0$. Moreover, $w_s > \bar{w}_s$ implies that firm L's profits at any other interior critical point ($h_s \in \mathcal{H}_s$) is less than what the firm expects to ear by choosing $h_s = 0$ and hence, that $h_s = 0$ must be the unique solution to firm L's problem (5). □

Proof of lemma 3. From the proof of lemma 2, $MB_s(0) > w_s$ is equivalent to,

$$\frac{2q_L(0)}{3}\left[\phi_E'(0)+[5\phi_E(0)-4\phi_L(h_p^*(0))-V]\sigma'(0)\right] > w_s$$

Because $V > 5\phi_E(0) - 4\phi_L(h_p^*(0))$ then, this inequality holds whenever,

$$|\sigma'(0)| > \frac{9w_s+6q_L(0)|\phi'(0)|}{2q_L(0)(V-5\phi_E(0)+4\phi_L(h_p^*(0)))}$$

which ensures that $h_s = 0$ is not optimal for firm L. Moreover, because $w_s < \bar{w}_s$, then any solution to firm L's problem must belong to $\mathcal{H}_S$. □

Proof of lemma 4. After some algebra, we obtain,

$$\Delta q_L(N) \equiv q_L(h_s, h_p, N+1) - q_L(h_s, h_p, N) =$$
$$\frac{2\sigma(h_s)(1-\sigma(h_s))[\sigma(h_s)(V-\phi_L(h_p)) - 2(V-\phi_E(h_s))]}{[4+4\sigma(h_s)(N-1)-\sigma^2(h_s)N][4+4\sigma(h_s)(N-2)-\sigma^2(h_s)(N-1)]}$$

Because $f(k) = 4 + 4\sigma(k-2) - \sigma^2(k-1) > 0$ for all $k \geq 2$, then the denominator of $\Delta q_L(N)$ must be strictly positive. Moreover, the numerator must be zero if either $\sigma = 0$ or $\sigma = 1$. In all other cases (i.e., for $0 < \sigma < 1$), the numerator must be strictly negative because the term within square brackets is the negative of the numerator of $q_L(2)$, which we have assumed strictly positive. □

Proof of lemma 5. By Lemma 9 in appendix B, $z_s(h_p, N)$ is (weakly) decreasing in N when $\phi_L(h_p) \geq \phi_E(h_s, \xi)$. Combining this with the first–order condition (12) and the strict convexity of ϕ_L (which makes $|\phi_L'|$ strictly increasing in h_p) gives (i) in the lemma. If $\phi_L(h_p) < \phi_E(h_s, \xi)$, the sign of $z_s(N+1) - z_s(N)$ is ambiguous, which gives (ii). □

Proof of lemma 6. From lemma 9,

$$z_s(h_p, N) = q_L(h_s, h_p, N)\left|\frac{\partial q_L(h_s, h_p, N)}{\partial \phi_L}\right|$$

where we write,

$$R(h_s) = \frac{2 + 2\sigma(h_s)(N-2)}{4 + 4\sigma(h_s)(N-2) - \sigma^2(h_s)(N-1)}$$

Therefore,

$$\frac{\partial z_s(h_p, N)}{\partial h_s} = \frac{\partial q_L}{\partial h_s} R(h_s) + q_L(h_s, h_p, N)\frac{\partial R(h_s)}{\partial h_s} \tag{24}$$

Because $\sigma(\bar{h}) = 0$ then:

$$q_L(\bar{h}, h_p, N) = \frac{V - \phi_L}{2}$$

and $R(\bar{h}) = 1/2$. Furthermore, $\frac{\partial R(h_s)}{\partial h_s} = \frac{dR(h_s)}{d\sigma}\frac{d\sigma(h_s)}{dh_s}$. Evaluating each of these terms at $\sigma(\bar{h}) = 0$ returns:

$$\left.\frac{dR(h_s)}{d\sigma}\right|_{\sigma=0} = \frac{8(N-2) - 8(N-2)}{16} = 0$$

and therefore, $\frac{\partial R(\bar{h})}{\partial h_s} = 0$ because $\sigma' < 0$. Thus, the sign of (24) is the same as the sign of $\frac{\partial q_L}{\partial h_s}$. Moreover,

$$\left.\frac{\partial q_L(h_s, h_p, N)}{\partial \sigma}\right|_{h_s=\bar{h}} = -\frac{(N-1)(V-\phi_E(\bar{h}))}{4}$$
$$< 0$$

because $V > \phi_E(\bar{h}, \xi)$ and $N \geq 2$. Therefore, $\frac{\partial q_L}{\partial h_s} > 0$ (because $\frac{\partial \sigma(h_s)}{\partial h_s} < 0$) and hence, $\left.\frac{\partial z_s(h_p,N)}{\partial h_s}\right|_{h_s=\bar{h}} > 0$. Finally, by continuity of the cross-partial derivative with respect to h_s, if it is strictly positive at $h_s = \bar{h}$, then it must remain positive for h_s in some interval $[\hat{h}_u, \bar{h}]$ for some $\hat{h}_u < \bar{h}$. Within this interval, h_p and h_s are strategic complements regardless of N. □

Proof of lemma 7. As in the proof of lemma 6,

$$\frac{\partial z_s(h_p, N)}{\partial h_s} = \frac{\partial q_L}{\partial h_s} R(h_s) + q_L(h_s, h_p, N)\frac{\partial R(h_s)}{\partial h_s} \tag{25}$$

where,

$$R(h_s) = \frac{2 + 2\sigma(h_s)(N-2)}{4 + 4\sigma(h_s)(N-2) - \sigma^2(h_s)(N-1)}$$

Consider the first term in (25). Because $\sigma' < 0$, and $R(0) = \frac{2}{3}$, the sign of this term depends on the sign of $\frac{\partial q_L}{\partial \sigma}$ evaluated at $\sigma = 1$:

$$\left.\frac{\partial q_L}{\partial \sigma}\right|_{h_s=0} = \frac{2N(V-\phi_L(h_p)) - (N+3)(V-\phi_E(0,\xi))}{9(N-1)}$$

The numerator of this expression is strictly positive under the condition in the lemma, and hence, the first term is undoubtedly negative under this condition. Regarding the second term in (25), note that $q_L(0, h_p, N) > 0$ and,

$$\frac{\partial R(h_s)}{\partial h_s} = \frac{2N}{9(N-1)}\sigma'(h_s) < 0$$

because $N \geq 2$ and $\sigma' < 0$. Therefore, the second term in (25) is strictly negative, and therefore, $\frac{\partial z_s(h_p,N)}{\partial h_s} < 0$ at $h_s = 0$ provided that the condition in the lemma holds. By continuity, if the cross-partial derivative is strictly negative at $h_s = 0$, it must remain negative in some interval $[0, \hat{h}_l]$ for some $\hat{h}_l > 0$. Within this interval, h_p and h_s are strategic substitutes. □

Proof of lemma 8. Recall that,

$$\frac{\partial \Pi_L(h_s, h_p, N)}{\partial h_p} = 2\underbrace{q_L(h_s, h_p, N)\left|\frac{\partial q_L}{\partial \phi_L}\right|}_{z_s(h_p,N)}\phi_L'(h_p)$$

and hence,

$$sign\left(\frac{\partial^2\Pi_L(h_p,h_s,N)}{\partial h_s\partial h_p}\right)=sign\left(\frac{\partial z_s(h_p,N)}{\partial h_s}\right)$$

where,

$$\frac{\partial z_s(h_p,N)}{\partial h_s}=\frac{\partial q_L}{\partial h_s}R(h_s)+q_L(h_s,h_p,N)\frac{\partial R(h_s)}{\partial h_s}$$

and,

$$R(h_s)=\left(\frac{2+2\sigma(h_s)(N-2)}{4+4\sigma(h_s)(N-2)-\sigma^2(h_s)(N-1)}\right)$$

From the proof of lemma 7,

$$\left.\frac{\partial z_s(h_p,N)}{\partial h_s}\right|_{h_s=0}=$$

$$\left[\frac{2}{3}\left(\frac{2N(V-\phi_L(h_p))-(N+3)(V-\phi_E(0,\xi))}{9(N-1)}\right)+q_L(0,h_p,N)\left(\frac{2N}{9(N-1)}\right)\right]\sigma'(0)$$

and hence,

$$\lim_{N\to\infty}\left.\frac{\partial z_s(h_p,N}{\partial h_s}\right|_{h_s=0}=\left[\frac{4(V-2\phi_L(h_p)+\phi_E(0,\xi))}{27}\right]\sigma'(0)<0$$

where the inequality follows because $\phi'<0$ and $V-2\phi_L(\cdot)+\phi_E(\cdot,\xi)>0$ from non-negativity of Cournot quantities. Therefore, there must exist some N^{**} such that for all $N>N^{**}$, human capital invested in technological and socially oriented innovations behaves as strategic substitutes near the point of perfect substitutability $h_s=0$). □

Proof of Proposition 3. From the text,

$$\Pi_L^*(0)=\max_{h_p\in[0,\bar{h}]}\left\{\left(\frac{V-2\phi_L(h_P))+\phi_E(0,\xi)}{3}\right)^2-w_ph_p\right\}$$

and,

$$\Pi_L^*(\bar{h})=\max_{h_p\in[0,\bar{h}]}\left\{\left(\frac{V-\phi_L(h_p)}{2}\right)^2-w_ph_p\right\}$$

The condition in the lemma can be rewritten as:

$$\Pi_L^*(\bar{h})-w_s\bar{h}>\Pi_L^*(0)$$

whenever $w_s<w_s^*$. Furthermore, non-negativity of Cournot quantities guarantee that $\frac{V-\phi_L(h_p)}{2}>\frac{2(V-\phi_L(h_p))-(V-\phi_E(0,\xi))}{3}$ and hence, $\Pi_L^*(\bar{h})>\Pi_L^*(0)$. Moreover,

$$\Pi_L^*(h_s)=\max_{h_p\in[0,\bar{h}]}\left\{\left(\frac{V-2\phi_L(h_p)+\phi_E(h_s)}{4-\sigma(h_s)}\right)^2-w_ph_p\right\}$$

and hence, $\Pi_L^*(h_s) \le \Pi_L^*(0)$ for any $h_s \in [0, \bar{h})$ because $\sigma(\cdot) \in [0, 1]$. Therefore:

$$\Pi_L^*(h_s) \le \Pi_L^*(0) < \Pi_L^*(\bar{h}) \quad \text{for all } h_s \in [0, \bar{h})$$

and hence,

$$\Pi_L^*(h_s) - w_s h_s \le \Pi_L^*(0) < \Pi_L^*(\bar{h}) - w_s \bar{h}$$

whenever $w_s < w_s^*$, which shows that $h_s = \bar{h}$ yields a strictly higher limiting profit than any other $h_s \in [0, \bar{h}]$. Consequently, given that the choice set $[0, \bar{h}]$ is compact, $\Pi_L(h_s, N) = [q_L^*(h_p, h_s, N)]^2 - w_p h_p$ converges pointwise to $\Pi_L^*(h_s)$, and the limit function $\Pi_L^*(h_s)$ possesses a unique maximizer,

$$\lim_{N \to \infty} h_s^*(N) = \bar{h}$$

as claimed. □

Proof of proposition 4. From the text, if $\sigma(h_s) \le \hat{\sigma}(h_p^B)$ then $q_L^B(h_p^B, h_s)(2 - \sigma^2(h_s)$ must be weakly lower than $2q_L^C(h_p^B, h_s)$. If we denote by $MB_p^B(h_p, h_s)$ the first term in the FOC (18), and $MB_p^C(h_p, h_s)$ this same term in FOC (19), the previous inequality implies $MB_p^B(h_p^B, h_s) \le MB_p^C(h_p^B, h_s)$. Moreover, as h_p^B and h_p^C are both interior solutions both $MB_p^B(h_p, h_s)$ and $MB_p^C(h_p, h_s)$ must be decreasing around these optimal values and hence, $h_p^B < h_p^C$. The same reasoning allows us to conclude that $h_p^C < h_p^B$ whenever $\sigma(h_s) > \hat{\sigma}(h_p^C)$. □

Proof of proposition 5. From condition $A_L'(0) > A_L^B$, $h_p^B(\cdot)$ must be interior. Moreover, from condition (20), $\frac{\partial^2 \Pi_L^B(h_s, h_p)}{\partial h_p^2} < 0$. Therefore, the implicit function theorem implies:

$$\frac{dh_p^B}{dh_s} = -\frac{\frac{\partial^2 \Pi_L}{\partial h_s \partial h_p}}{\frac{\partial^2 \Pi_L}{\partial h_p^2}}$$

and the sign of this expression is the same as the sign of the cross-partial derivative of firm L's profit function. For ease of notation, denote by MB_p^B the first term in FOC (18):

$$MB_p^B = \frac{2\phi_L(h_p) A_L'(h_p)}{(4 - \sigma^2(h_s)) A_L(h_p)} \left[q_L^B(h_p, h_s)(2 - \sigma^2(h_s)) \right]$$

Then,

$$\frac{\partial^2 \Pi_L^B}{\partial h_s \partial h_p} \equiv \frac{\partial MB_p^B}{\partial h_s} = \frac{\partial MB_p^B}{\partial \phi_E} \frac{\partial \phi_E}{\partial h_s} + \frac{\partial MB_p^B}{\partial \sigma} \frac{\partial \sigma}{\partial h_s} \tag{26}$$

Because $\frac{\partial \phi_E}{\partial h_s} < 0$ and,

$$\frac{\partial MB_p^B}{\partial \phi_E} = \frac{2\phi_L(h_p)(2 - \sigma^2(h_s))\sigma(h_s) A_L'(h_p)}{(4 - \sigma^2(h_s))^2 (1 - \sigma^2(h_s)) A_L(h_p)} \ge 0$$

the first term in (26) is non-positive. Moreover, $\sigma'(\cdot) < 0$ and hence, the sign of expression (26) depends on the sign of:

$$\frac{\partial MB_p^B}{\partial \sigma} = \frac{2\phi_L(h_p)A_L'(h_p)}{A_L(h_p)(4-\sigma^2(h_s))^2}\left[\frac{\partial q_L^B}{\partial \sigma}(2-\sigma^2(h_s))(4-\sigma^2(h_s)) - 4\sigma(h_s)q_L^B(h_s,h_p)\right]$$

Define $N(x) = V(2-x-x^2) - a(2-x^2) + xb$, $D(x) = (1-x^2)(4-x^2)$, and $f(x)$ by:

$$f(x) = (2-x^2)(4-x^2)\left[N'(x)D(x) - D'(x)N(x)\right] - 4xN(x)D(x)$$

Clearly, $f(x)$ is continuous, with $f(0) = -32(V-b)$, and $f(1) = 18(b-a)$. Thus, provided that $V > b$ and $b > a$, we have $f(0) < 0$ and $f(1) > 0$, which would imply that $f(x)$ crosses zero at least once. Moreover, continuity of $f(x)$ would guarantee the existence of some strictly positive $\hat{x}$ satisfying $\hat{x} < 1$ such that $f(\hat{x}) < 0$.

Let $a = \phi_L(h_p^*(h_s))$, $b = \phi_E(h_s)$, and $x = \sigma(h_s)$. Then,

$$\frac{\partial MB_p^B}{\partial \sigma} = \frac{2\phi_L(h_p)A_L'(h_p)}{A_L(h_p)(4-\sigma^2(h_s))^3(1-\sigma^2(h_s))}f(\sigma(h_s))$$

Let $\hat{h}$ be implicitly defined by $\sigma(\hat{h}) \equiv \hat{x}$. Because $\hat{x} \equiv \sigma(\hat{h}) < 1$ and $\sigma' < 0$, then $\hat{h} > 0$. Moreover, the conditions in the proposition ensure that $h_p^* > 0$ and thus, continuity of ϕ_E ensures existence of some $0 < \underline{h} < h_p^*$ such that $\phi_E(\underline{h}, \xi) > \phi_L(h_p^*)$ holds. Therefore, $f(\sigma(\underline{h})) > 0$ and $\frac{\partial MB_p^B}{\partial \sigma} > 0$ near $h_s = 0$, whereas $f(\sigma(\bar{h}) \equiv f(0) < 0$ then $\frac{\partial MB_p^B}{\partial \sigma} < 0$ near $h_s = \bar{h}$.

Finally, define

$$\Delta(h_p^*) = \left\{ h_s \in (0, \bar{h}] : \frac{\partial^2 \Pi_L^B(h_s, h_p^*(h_s))}{\partial h_s \partial h_p} = 0 \right\}$$

Because this cross partial is continuous, strictly positive at $\bar{h}$, and negative for h_s near zero, then $\Delta(h_p^*)$ is non-empty. Thus, letting $h_u = sup_{h_S \in (0,\bar{h}]}\Delta(h_p^*)$ and $h_l = inf_{h_s \in (0,\bar{h}]}\Delta(h_p^*)$ we must have $0 < h_l \leq h_u < \bar{h}$, and h_p and h_s behaves as substitutes for all $h_s \in (0, h_l)$, and as complements for $h_s \in (h_u, \bar{h}]$. □

Proof of Proposition 6. Because $w_s < \bar{w}_s$ then there must exists some $h_s^* \in (0, \bar{h}]$ such that

$$\bar{\Pi}_L^B(h_s^*, h_p^*(h_s^*)) - w_s h_s^* > \bar{\Pi}_L^B(0, h_p^*(0))$$

and hence, firm L must obtain a strictly higher profit choosing some $h_s^* > 0$ instead of $h_s = 0$ regardless of the decision regarding technological innovation $h_p^*(\cdot)$ in the second stage. □